"TELESCOPE" for Triangle Geometry. Grand Theorems

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Abstract

A program has been compiled that searches for Theorems "by induction" from zero to infinity.

Telescope for astronomy (history)

On January 6, 1610, Galileo sat at home and fiddled with a product that he had assembled himself: a meter-sized telescope. This night, for the first time, he will direct it upwards to the stars and the moon. What does he see in her?

a)   Everyone else knows!

Court of the Inquisition, sentence and great posthumous glory to the Man who discovered the SCIENCE of ASTRONOMY! Because modern astronomy does not exist at all without telescopes (Figure 1).

Figure 1: Galileo Galilei and his discoveries. Left: what was known BEFORE 01/07/1610; right: what happened AFTER.

Telescope for Triangle Geometry

The most important thing in this science is that any of its statements can be easily verified directly. Example: a drawing is drawn - it is required to prove that for any triangle the area S1 is equal to S2. We are demonstrating! We draw two (yes, only 2! .. no more!) different triangles on the computer. Calculate on the same computer for two cases two areas: S1 and S2. We determine their equality up to the seventh digit inclusive. And we draw an unambiguous conclusion about the reliability of this fact! Who will argue with this? (This circumstance, by the way, fundamentally distinguishes this science from, say, the Theory of Numbers! Suppose that we have formulated a certain position for any natural

N. We have tested it on a very large number of first numbers (on a computer). But nothing follows from this, and it may be that for some VERY large number this provision will be violated!) Let's describe the device of our TELESCOPE for TG!

More about Triangle Centers

In the triangle there are so-called Triangle Centers (point of intersection of medians; heights, etc.) The school course speaks of the first few, but there are actually 50,730 of them. (For a complete list, see Clark Kimberling's Encyclopedia of Triangle Centers [1] !) On Figure 2 we have depicted eight of them. Take some set of these points and call them the base points.

Subsequent Iteration and Final Diagram

At the first stage, straight lines are drawn through all pairs of base points. All points of intersection between them are found. (If the coordinates of the points are the same, only the first option is taken, the rest are ignored). The resulting points are numbered and called ZERO Iteration points [2]. After that, all four points of the Zero Iteration are taken and those of them that lie on the same circle are determined. (Table 1) shows the approximate number of checks required for this). Then all the points of the ZERO Iteration are taken, and the points of the FIRST Iteration are added to them ... and in the same way as in the case of the first iteration, the points of the SECOND Iteration are determined. Further, according to the principle of mathematical induction ... until new centers are exhausted, or the computer resource runs out! All results are graphically displayed on the diagram (Figure 3c). That's all! This is an exhaustive description of our TELESCOPE [7]! (Though there is one small addition, which is explained in chapter 6).
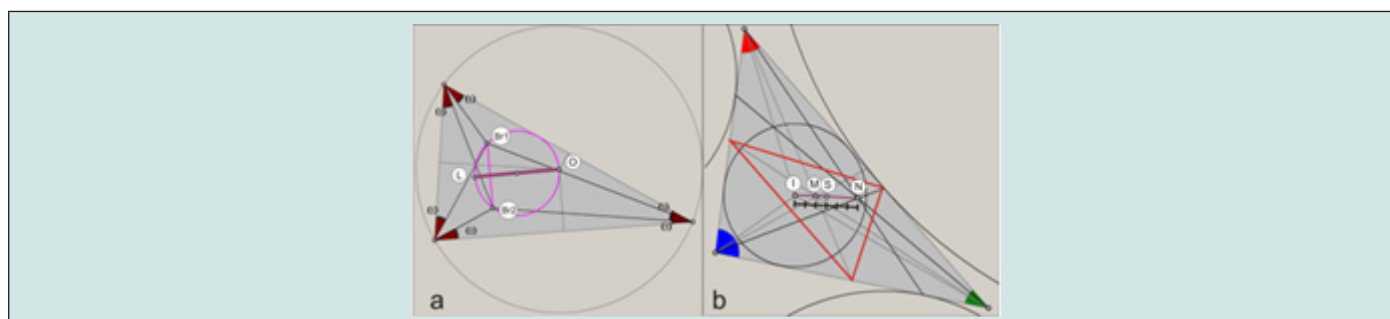

Figure 2: a-four Triangle centers of the "Brocard butterfly" (O – orthocenter; L – Lemoine point[2]; Br1-2 – Brocard points[3]); 2b - four more singular points (intersection point of the bisectors (I), medians (M), bisectors of the "median triangle" (red - in Fig. 3b; Spieker's point[4]; (S)) and Nagel's point[5] (N)) lying at quantized distances, on the Nagel line[6].
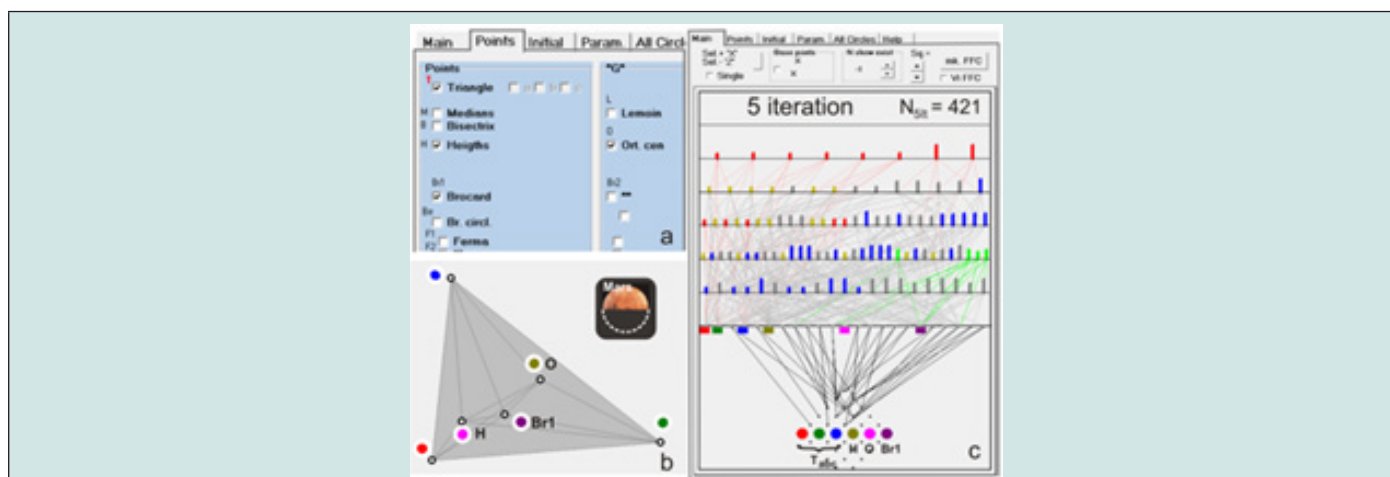

Figure 3: a – panel for selecting base points; 3b is a view of these points on the control canvas (why there is an image of Mars here and why there is only one hemisphere, will be explained later); 3c – the result of the program (final Diagram).

Table 1: Their centers are determined. They are called the FIRST Iteration points.

T (number of base points)	L (number of lines)	N (number of intersections)	P (number of potential circles)
5	(5*4)/2 = 10	(10*9)/2 = 45	~ 444 = 3.7*106
6	(6*5)/2 = 15	(15*14)/2 = 105	~ 1044 = 1.0*108

7	(7*6)/2 = 21	(21*20)/2 = 210	∼ 2094 = 1.6*109
8	(8*7)/2 = 28	(28*27)/2 = 375	∼ 3774 = 2.9*1010
9	(9*8)/2 = 36	(36*35)/2 = 630	∼ 6294 = 1.5*1011

How the Diagram works

The first thing to understand... Each element of our Diagram (multi-colored bar in Figure 3c) is just a RECORD how to quickly find the position of the center of the given circle from any position of the vertices of the triangle (and hence its radius; since the centers of the four "subordinate" circles on which it rests are known). Let's select and click the mouse (left button) on some element – bars (circle number 77) on the Diagram ("Main" tab; Control Panel-2).

All characteristics of a given circle will appear on the Control Panel-1 (Figure 4a). Let's put the "Single" switch at the top, on the Control Panel-2 ("Main" tab) to the YES position. Only one selected element will be highlighted on the Diagram (Figure 5). Let's put the "Manual" switch in the upper left corner. On the original triangle, a small circle will appear around its vertex "C". Now click with the left mouse in this circle and move the vertex "C" ANYWHERE! The program in the "online" mode will redraw both the triangle itself and your selected circle.
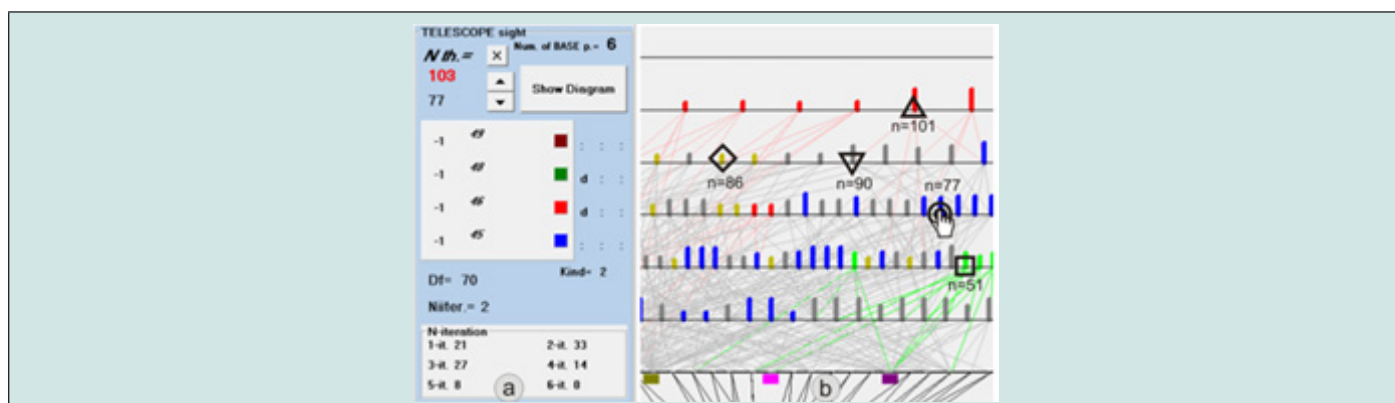

Figure 4: a - characteristics of circle number 77. Four more selected (Fig. 4b) circles are used below as examples.
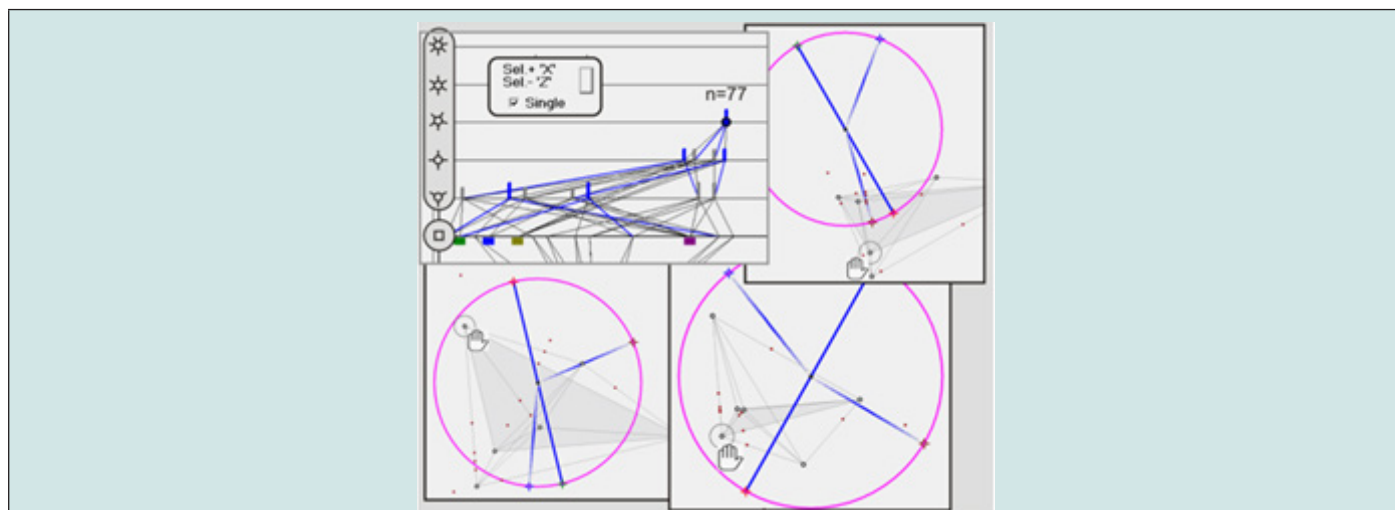

Figure 5: Demonstration of the correct operation of our program (the TELESCOPE)!

"Complexity" of circles. Bars height

How to determine the height of a bar in a Diagram? Each of our circles (in fact, records; bars) corresponds a certain "complexity" D (see Figure 6). This is a number that determines the sum of the complexities of all four circles on which it relies. (The complexity of base points is assumed to be 1. The complexity of the circle in the first iteration is the total number of points it took to draw it).Now for the little addition we talked about earlier. When our centers coincide in the process of drawing the Diagram, we choose for the Diagram the circle (record) with the smallest D! Let's try to explain our choice now... The coincidence of our centers is, as it were, a new Theorem. There are usually quite a lot of them, but they, as a rule,

fall on the some chosen center. It's like a bumps hanging on our tree trunk [3]. You can explore them further! But we are interested only in the trunk itself. That is, all centers of our circles lie in fundamentally new places. The height of each bar just depends on

D and is determined by the formula in Figure 7. (The minimum and maximum D for this iteration is determined at the beginning. And then the formula on the right is used).
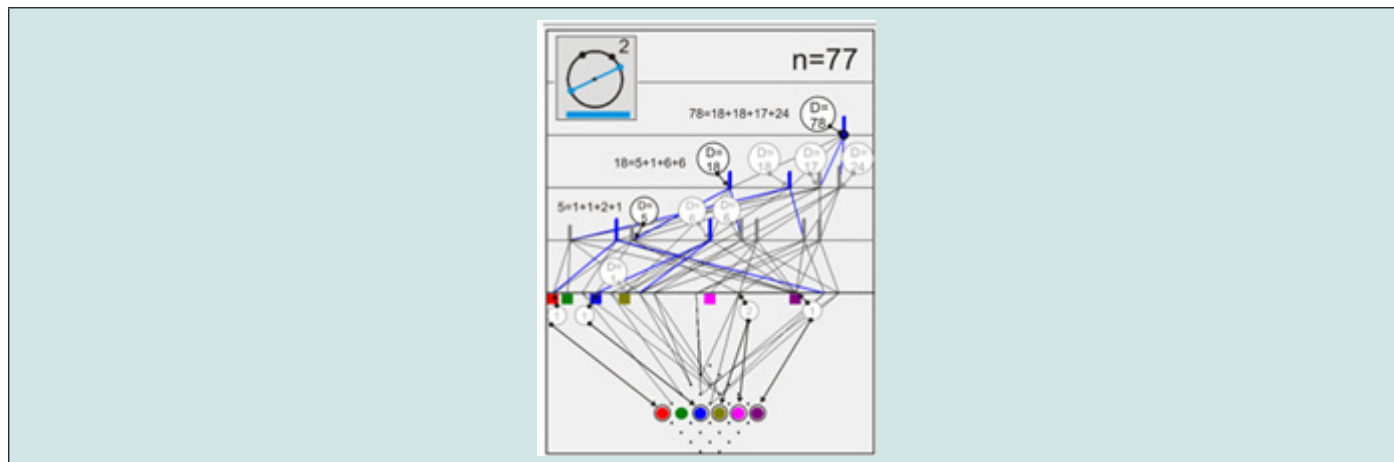

Figure 6: shows how the finite complexity is calculated for the circle number 77 depicted in figures 4b and 5.
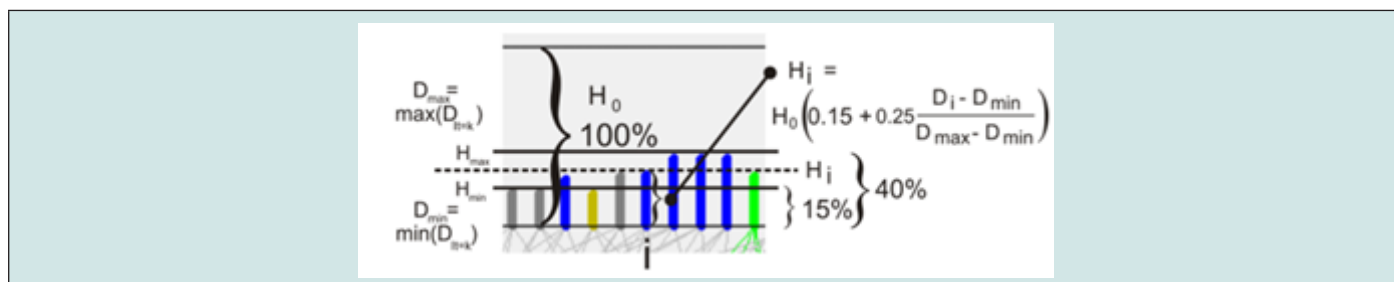

Figure 7: shows how the height of our bar is calculated depending on its "complexity" (D).

"Kind" of circles. Our colors

In total, there can be more than 10 different arrangements of four points on a circle. Figure 8 shows how they are colored on the Diagram (as a bar) and on the Control Canvas. All "kinds" are numbered. You can think of more locations, but that's enough for

us! Figure 9 shows how our Program draws various "kinds" of circles. (The circle itself is always painted pink - pink (Fuchsia). All other "kinds" are drawn in a strictly defined color; see Fig. 8. All radii are drawn in the same color as "symmetries" but using a gradient – the "color" rises to its values within the half of radius, and then falls back) [4,5].
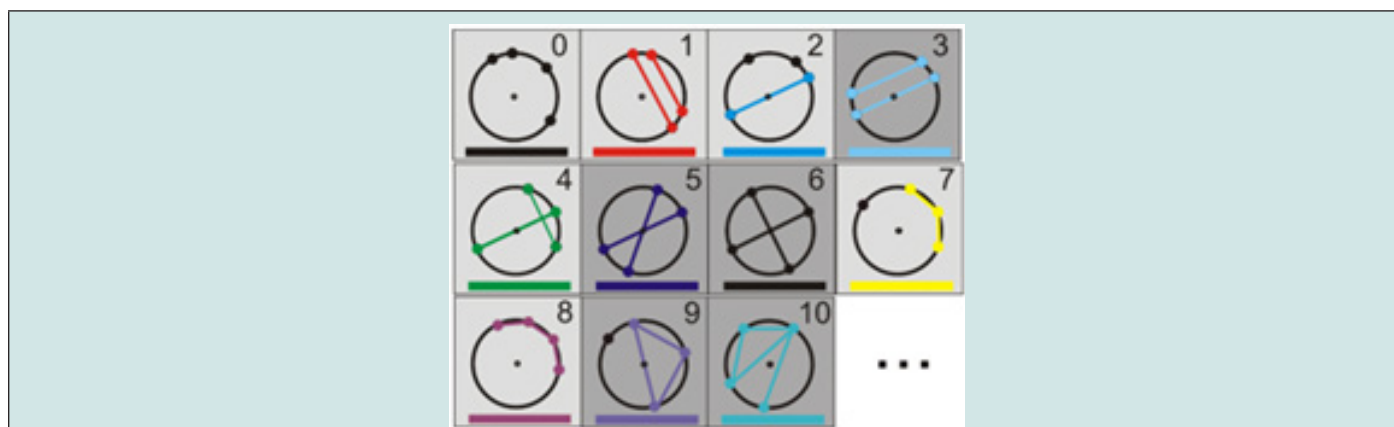

Figure 8: "Kinds" (K); the essence is the location of four points on a circle. (The darkened numbers have not yet been encountered by the author).

First observations with the help of our TELESCOPE. GRAND theorems

If we take a set of random base points: the anti-bisector, Gergon's point, Specker's point, etc., etc., and point our telescope at them, the answer is total blackness. "Zero circles" already in the first iteration! And therefore, in all subsequent! But there are rare, interesting sets in which everything goes differently. In which the number of circles increases from iteration to iteration! Moreover, some sets do it so quickly that the memory of my home computer is already overflowing on the first few iterations... and one can only guess what will happen next! We will say that such sets form a GRAND theorem.

Let's give the first simple definition of what a GRAND theorem is! A given set of initial points forms a GT if its output Diagram contains at least FIVE first iterations and the total number of circles in it is greater than 100! An example in Fig. 3 – set of base points T(ABC), H, O, Br1 – satisfies this condition. What did we see when we first looked through our Telescope? Let's continue the analogy with astronomy! Galileo observed primarily the planets. They are known to be divided into three groups. The first group is a group of earth-like planets: Mercury, Venus, Earth, Mars. Group of gas giants: Jupiter, Saturn, and finally, a group of ice giants: Uranus, Neptune.

From the very beginning, we also see three GRAND theorems, or rather, three cores for GRAND theorems.

Let's call the core of the first - "Mars" (planets from the first group). It consists of 7 base points: the vertices of the triangle themselves A, B, C; point of intersection of heights, orthocenter, and two points of Brocard (Figure 10a). The probability of forming a GRAND theorem increases if there is a "natural" equilateral triangle among the base points. Therefore, the core of the second GRAND theorem – let's call it "Uranus" – consists of 6 main points: the vertices of the triangle T: A, B, C; and the vertices of the "trisector" (Morley's equilateral triangle). The core of the third GRAND theorem - let's call it "Jupiter" – consists of two equilateral Napoleon triangles. (We take only them as base points, without the vertices of the Triangle itself). The number of base points in this case is also six. Let's build Diagrams for all three cases and start with the "Mars" (Figure 11). We use the very first Diagrams as they look on the computer! Of course, the bars in each iteration must be grouped by using 3!=6 symmetries associated with the vertex permutations, etc. But this is our first review! Let's note on the Diagram of "Mars" two circles with numbers n=33, and n=139. One of them coincides with the center of the Euler circle, and the second with the center of the Brocard circle.
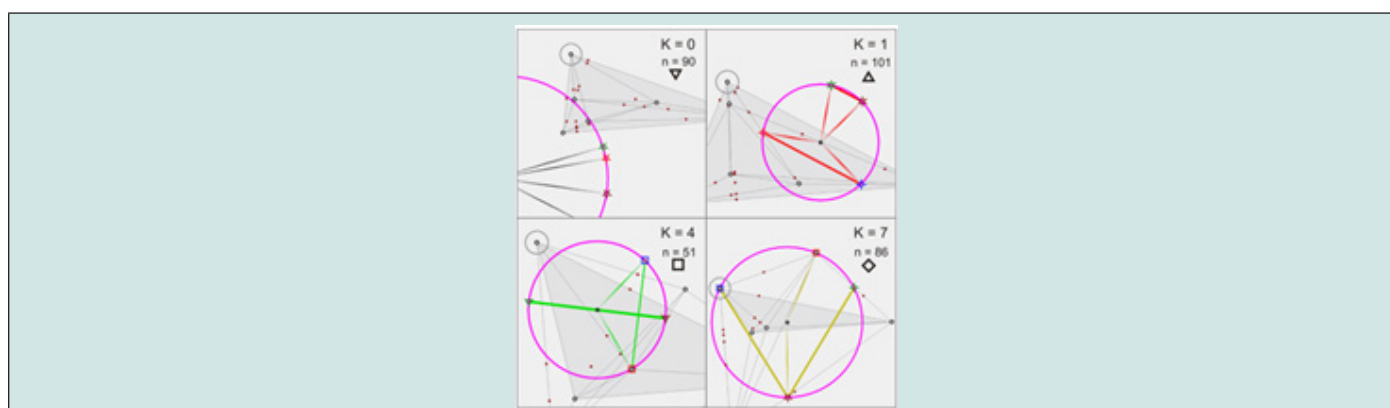

Figure 9: shows how the various "kinds" of circles looks on the Control Canvas. The circles are taken from Fig. 4b.
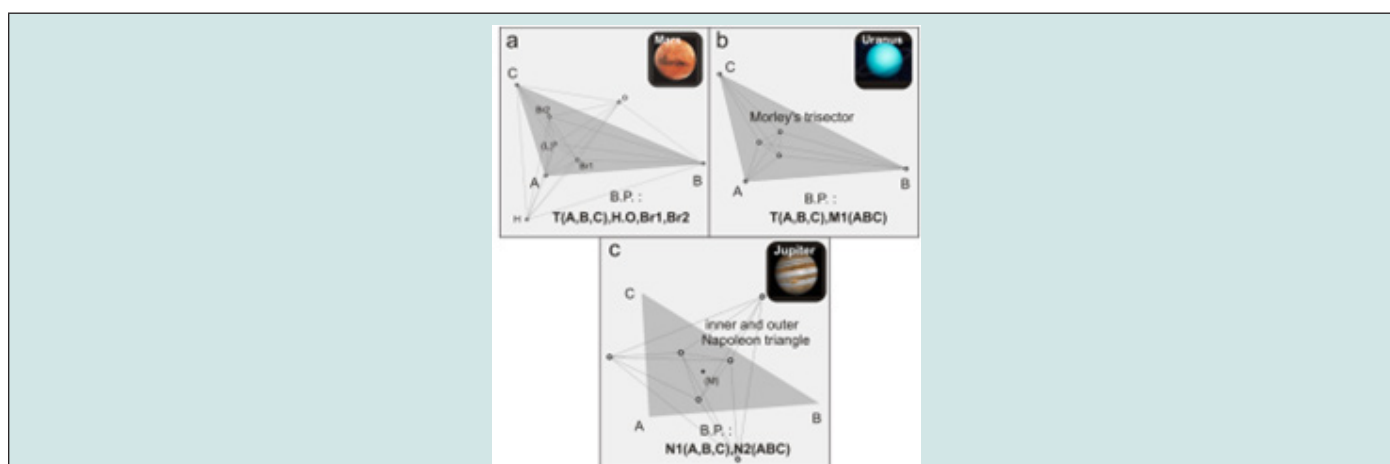

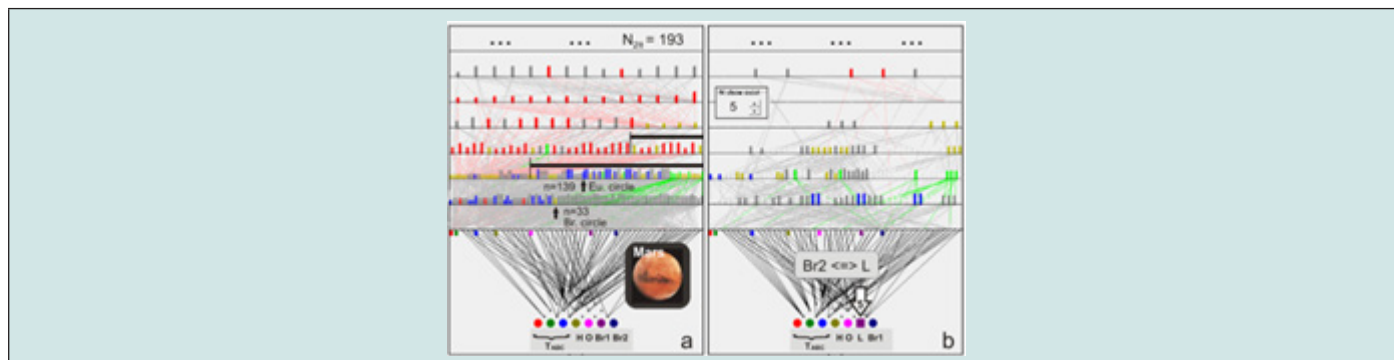**Figure 10:** Our first three "cores" for GRAND theorems.

Figure 11: Diagram of the GRAND theorem "Mars". We added to "Mars" (Fig. 3) the "south pole" – the second point (Br2) of Brocard – and the total number of circles changed radically! The diagram already at the second iterations becomes "not computable" on a home computer. Black lines in Figure 11: a are the curtains we are used to hide iteration areas due to lack of computer memory. On Fig. 11b, we replaced one of the Brocard points with the Lemoine point and again built our diagram of 6 iterations. We left visible only those circles that use the Lemoine point I n the construction. It can be seen that such circles exist.

On these centers lie hundreds and hundreds of "bumps" that we talked about in Chapter 6. ("Hundreds" is not an exaggeration. Suppose the program places ten points on the Euler circle. Then it interprets this as having C410 = 210 separate circles with the same center. Which all but one are discarded!) But, in the end, this strange "joint work" of the Euler circle and the Brocard circle leads to an EXPLOSIVE increase of the Diagram elements in the subsequent iterations [6]. Figure 12 shows diagrams of the GRAND theorems

for "Uranus" and "Jupiter"... It is interesting that simply adding one of two Fermi points to "Jupiter" leads to an explosive increase the number of circles already in the FIRST iteration (from 28 to 246)! But ... let's leave this conversation for now, and let's deal with the "cores". Looking at Figures 11 & 12 it is obvious how important the "KINDS" of our circles (K) are for the construction of our Diagrams, i.e. the colors of our "bars"!
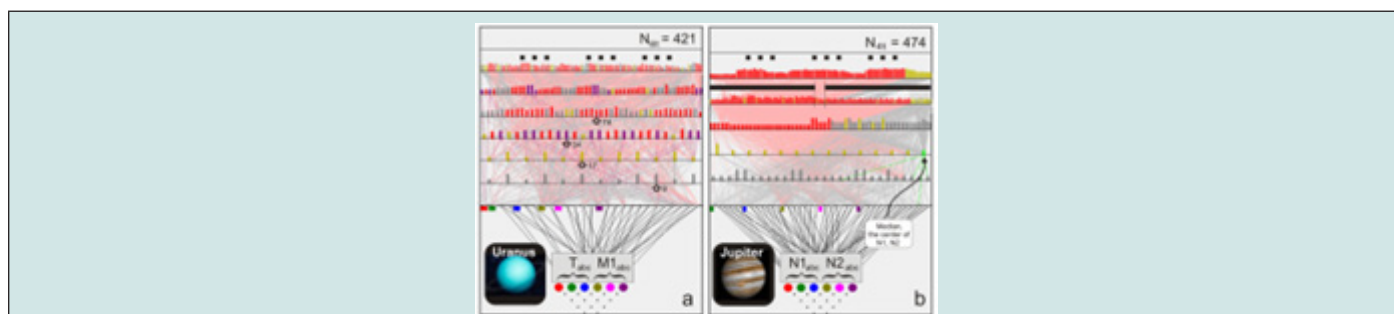

Figure 12: GRAND theorems for "Uranus" and "Jupiter".

Let's formulate the first three observations that can be made by looking at our Diagrams.

a) Each GRAND theorem is characterized by its own set of "kinds". All of our GRAND theorems have circles with K = 0, 1, 7 (gray, red and yellow). In the GRAND Theorem "Mars" the circles with K = 2 and 4 (blue and green) are added to them. In the GRAND theorem "Uranus" only the circles with K = 8 (violet) are added to the circles with K = 0, 1, 7.

b) If some kind of circle appeared at some iteration, then it will certainly appear further. (True for circles of the K = 2, 4, 8 in Figures 11 & 12).

c) All three GRAND theorems are characterized by

"redshifts", that is, as the number of iterations increases, more and more circles become "red" (kind K=1).

The program allows you to draw all the circles (with their colors depending on the "kinds") for each iteration on one image, while maintaining the ability to transfer the vertex with an arbitrary point. There is also a button to turn the original triangle into an equilateral one. Click on it and draw the circles for the first four iterations for "Uranus" (Figure 13). There is the question about the quality of our Diagrams? All bars exist in reality! The easiest way to check this: put the vertex "C" at an arbitrary point (see chapter 5); select the circle with n=0 and by pressing "X" (scroll) view all N circles in any of the Diagrams. But there is another interesting check related to the slight "corruption" of our original drawing [7,8].
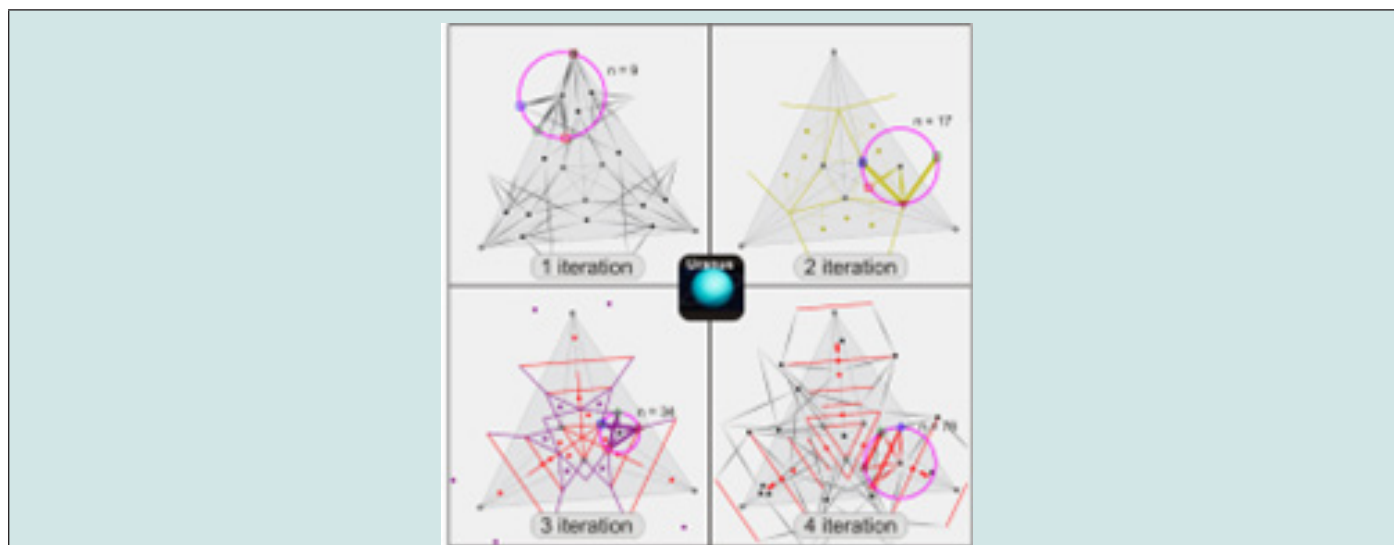
Figure 13: First four iterations for Uranus. (In figure 4, the "yellow" (K = 7) circles were not drawn. They complicate the already confusing image). In each of the four figures, we have highlight one of the circles from Fig. 4b.

Let's take a set from the GRAND theorems "Uranus", and in an equilateral triangle from it we will bring its vertices closer to the center by a small percentage of its original length. Let's see what will happen to our Diagrams. See Figure 14. The diagram almost completely disappears when the vertices approach the center by only 0.01% of the original length. On a home computer, this distance is approximately 40 pixels. 0.01% of 40 pixels is 0.004 of pixel size! It is basically indistinguishable to the human eye! Another question is "are all real circles present in the diagram?" The answer is NO! The radius of a particular circle may turn out to

be too large, so it will be rejected. Two vertices may be too close… and so on. How to deal with this will be discussed in future papers. In the last drawing (Figure 15), we have shown together all the centers of the circles for "Uranus" for iterations 1 - 5. Each center on the Diagram is represented by a circle. Its color is the color of the "kind" of the circle. Their radii decrease with increasing iteration, and each circle is intersected by a segment equal to three circles. The segment angle is chosen randomly each time. Now, if our two centers coincided with each other, this would immediately become clear. But that doesn't happen, so it's okay!
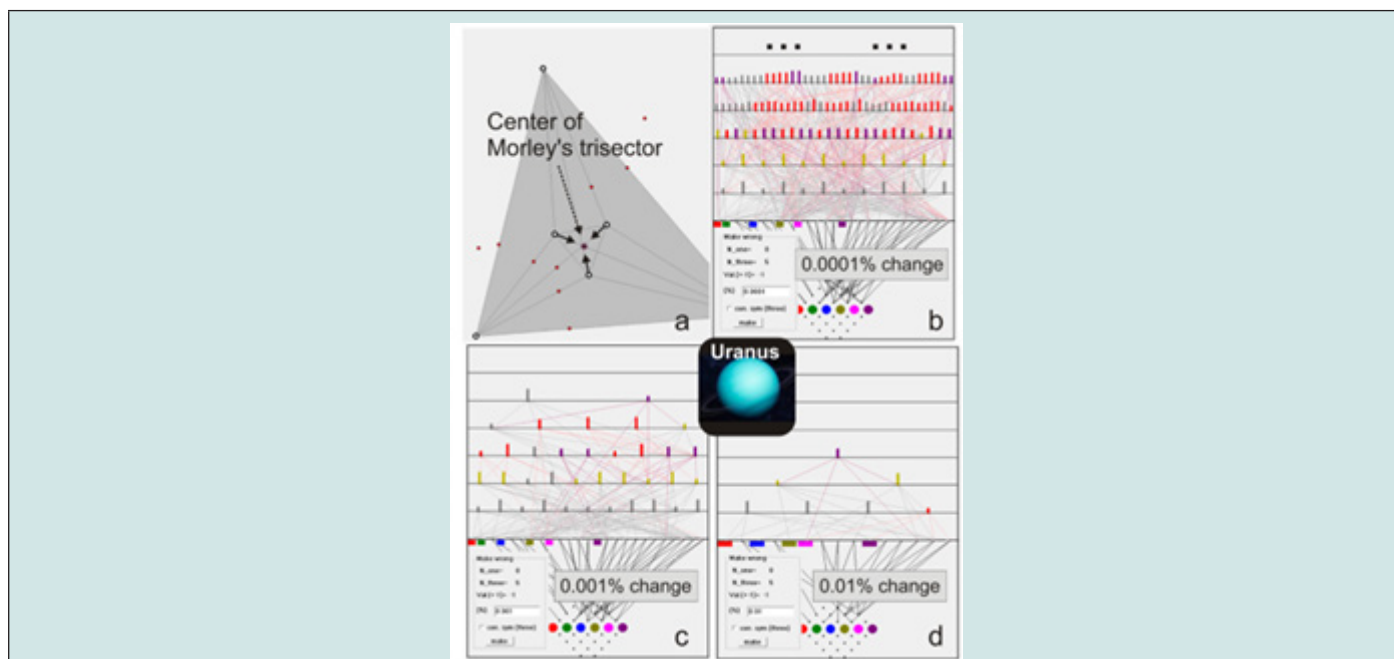

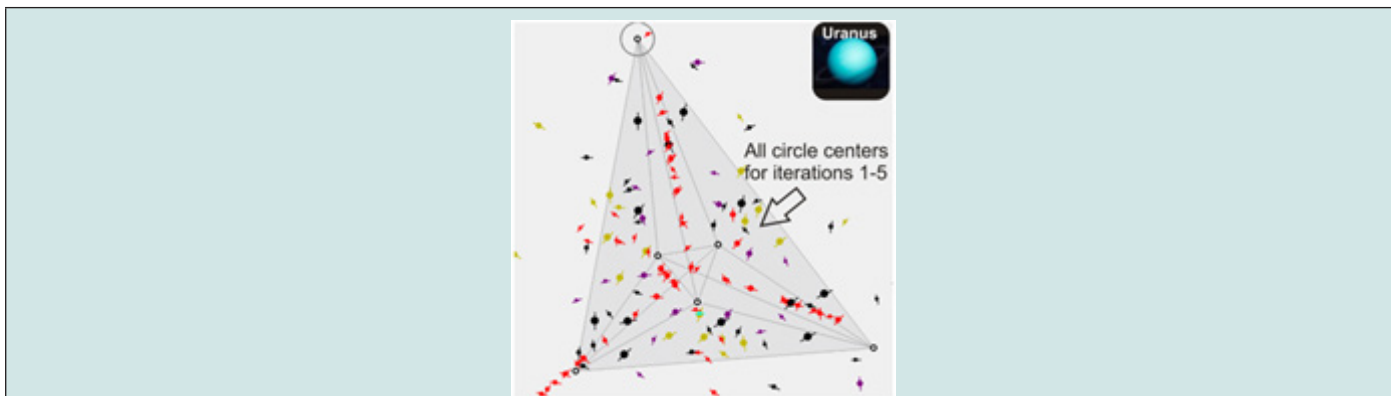**Figure 14:** The reaction of the program to the distortion of our drawing.

Figure 15: All the centers of the circles are shown in one figure. The strikethrough ensures that no two centers match!

Conclusion

So how many circles are there in the GRAND theorems at all iterations from zero to infinity? The real total number of circles in all three GRAND theorems is considered to be infinity! Which is indirectly confirmed by the fact that the number of new circles from iteration to iteration does not just increase, but increases with acceleration!

Summary

In the Triangle Geometry, there are not just Theorems, but entire GALAXIES from an infinite set of Theorems. Very beautifully and differently arranged and held together by some strange gravity. Mathematicians do not know this, which is connected with the peculiarities of human vision. But now, with the invention of the telescope, let's hope the things get better. In any case, the author continues to work on the device. He has yet to enter 50,730 triangle centers into the program.

References

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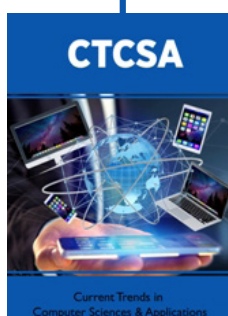
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