

Numerical Solution of Boundary Layer Flow of Viscous Fluid Via Successive Linearization Method

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Received: 📅 November 19, 2018; Published: 📅 December 04, 2018

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Abstract

The aim of this work is to obtain the numerical solutions for the boundary layer flow of heat transfer of incompressible viscous fluid. The governing partial differential equations are converted into ordinary differential equation by using a similarity transformation. The nonlinear equation governing the flow problem is modeled and then solved numerically by means of a successive linearization method (SLM). The numerical results are derived in tables for comparisons. The important result of this comparison is to show the high precision of the SLM in solving system of nonlinear differential equations. Graphical outcomes of various parameters such as Prandtl number (Pr) and Eckert number (Ec) on the flow, field are discussed and analyzed. Besides this the present results have been tested and compared with the available published results in a limiting manner and an excellent agreement is found.

Keywords: Viscous fluid; Successive linearization; Boundary layer

Introduction

In the recent years, a great deal of interest has been gained to fluids applications. Some fluids not easy to expressed by particular constitutive relationship between shear rates and stress and which is totally different than the viscous fluids [1,2]. These fluids including many home items namely, toiletries, paints, cosmetics certain oils, shampoo, jams, soups etc. have different features and are denoted by non-Newtonian fluids. In general, the categorization of non-Newtonian fluid models is given under three class which are named the integral, differential, and rate types [3-6]. In the present study, the main interest is to discuss the heat transfer flow of hydrodynamic viscous fluid over a flat plate in a uniform stream of fluid with dissipation effect. The most phenomena in the field of engineering and science that occur is nonlinear. With this nonlinearity the equations become more difficult to handle and solve. Some of these nonlinear equations can be solved by using approximate analytical methods such as Homotopy analysis method (HAM) proposed by liao S [7,8], Homotopy Perturbation method (HPM) it was found by Ji-Huan [9] and Adomain decomposition method (ADM) Q Esmaili et al. [10], Makinde OD et al. [11] and Makinde OD [12].

However, some of these equations are solved via traditional numerical techniques such as finite difference method, shooting

method and Keller box method, Runge-Kutta. Recently some studies have presented a new method called Successive Linearization Method (SLM). This method has been applied successfully in many nonlinear problems in sciences and engineering, such as the MHD flows of non-Newtonian fluids and heat transfer over a stretching sheet [13], viscoelastic squeezing flow between two parallel plates [14], two dimensional laminar flow between two moving porous walls [15] and convective heat transfer for boundary layer with pressure gradient [16,17]. Therefore, the effectiveness, validity, accuracy and flexibility of the SLM are verified among of all these successful applications. Presently a new investigation on the heat transfer flow of hydrodynamic viscous fluid over a flat plate in a uniform stream of fluid with dissipation effect is discussed. The numerical solution to the resulting nonlinear problem is computed by using the SLM approach. The embedded flow parameters are discussed and illustrated graphically.

Mathematical formulation of the problem

The governing equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$

$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{k}{c_p} \left(\frac{\partial u}{\partial y} \right)^2$ where (u,v) are the components of velocity in (x, y) directions, $\nu = \left(\frac{\mu}{\rho} \right)$ the kinematic viscosity T is temperature of fluid, $\alpha = \left(\frac{k}{\rho c} \right)$ the thermal diffusivity,

k the fluid thermal conductivity, ρc the fluid capacity heat and c_p the specific heat. The relevant boundary conditions are defined as

$$u = 0, T = T_w(x) \text{ at } y=0$$

$$u = U, T = T_\infty \text{ at } y \rightarrow \infty$$

Where T_w, T_∞ are constants. Introducing the following dimensionless variables

$$\eta = y \sqrt{\frac{U}{\nu x}}, \psi = \sqrt{U\nu x} f(\eta), u = \frac{\partial \psi}{\partial y} = U f'(\eta),$$

$$v = -\frac{\partial \psi}{\partial x} = \frac{1}{2} \sqrt{\frac{U\nu}{x}} (\eta f' - f), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$

Utilizing equation (6), equation (1) is satisfied automatically and equations (2) and (3) characterize to the following problems statement

$$f''' + \frac{1}{2} f f'' = 0 \quad (7)$$

$$\theta'' + \frac{1}{2} \theta' + \text{Pr Ec} (f'')^2 - \eta \text{Pr} f' \theta = 0 \quad (8)$$

The related boundary conditions

$$f' = 0, f = 0, \theta = 1 \text{ at } \eta = 0 \quad (9)$$

$$f'(\infty) = 1, \theta(\infty) = 0 \quad (10)$$

Solution of the problem

Here successive linearization method (SLM) [14-16] is implemented to obtain the numerical solutions for nonlinear system (8) and (10) corresponding to the boundary condition Eq. (11) - (13) (Table 1). The convergence for numerical values of $f''(0)$ and $-\theta'(0)$ for different order of approximation when $\text{Ec} = 0.01, \text{Pr} = 1$ and $n = 1.00$ (Table 1).

Table 1: The convergence for numerical values of $f''(0)$ and $-\theta'(0)$ for different order of approximation when $\text{Ec} = 0.01, \text{Pr} = 1$ and $n = 1.00$

Order of approximation	$f''(0)$	$-\theta'(0)$
1	0.3612452754	0.5821272282
5	0.3320573366	0.5412124295
10	0.3320573366	0.5412124295
20	0.3320573365	0.5412124295
30	0.3320573365	0.5412124295
50	0.3320573365	0.5412124295

The numerical values of $f(\eta)$ and $f'(\eta)$ when, $n = 1, \text{Pr} = 1$ for $\text{Ec} = 0.01$. (Table 2).

Table 2: The numerical values of $f(\eta)$ and $f'(\eta)$ when, $n = 1, \text{Pr} = 1$ for $\text{Ec} = 0.01$.

Ec	η	$f(\eta)$	$f'(\eta)$
0.1	0	0	0
	0.1	0.001648	0.033205
	0.3	0.014941	0.099599
	0.5	0.041467	0.165884
	1	0.165539	0.329774
	2	0.649981	0.629745
	3	1.396860	0.846097
	4	2.305718	0.955494
5	3.283272	0.991540	

The numerical values of $\theta(\eta)$ and $-\theta'(\eta)$ when, $n = 1, \text{Pr} = 1$ for $\text{Ec} = 0.01$ (Table 3).

Table 3: The numerical values of $\theta(\eta)$ and $-\theta'(\eta)$ when, $n = 1, \text{Pr} = 1$ for $\text{Ec} = 0.01$.

Ec	η	$\theta(\eta)$	$-\theta'(\eta)$
0.01	0	1.000000	0.541212
	0.1	0.945928	0.539719
	0.3	0.838986	0.527814
	0.5	0.735487	0.505875
	1	0.502477	0.420608
	2	0.186143	0.214775
	3	0.049496	0.074145
	4	0.009246	0.017244
5	0.001176	0.002656	

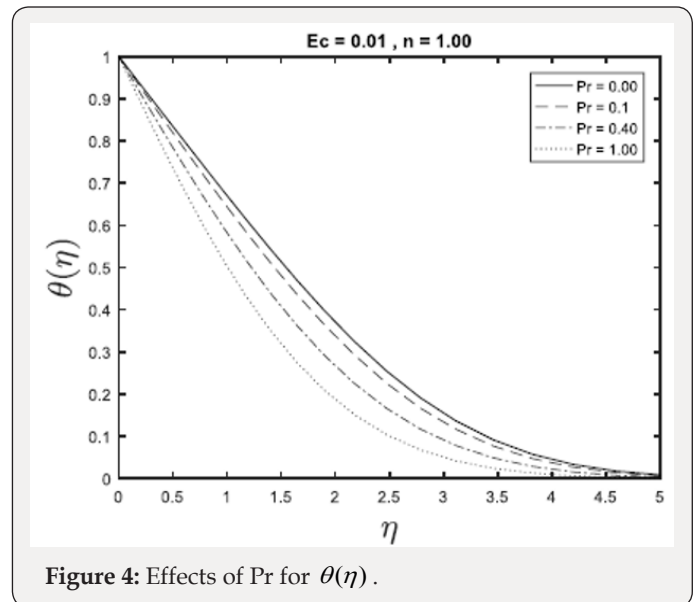
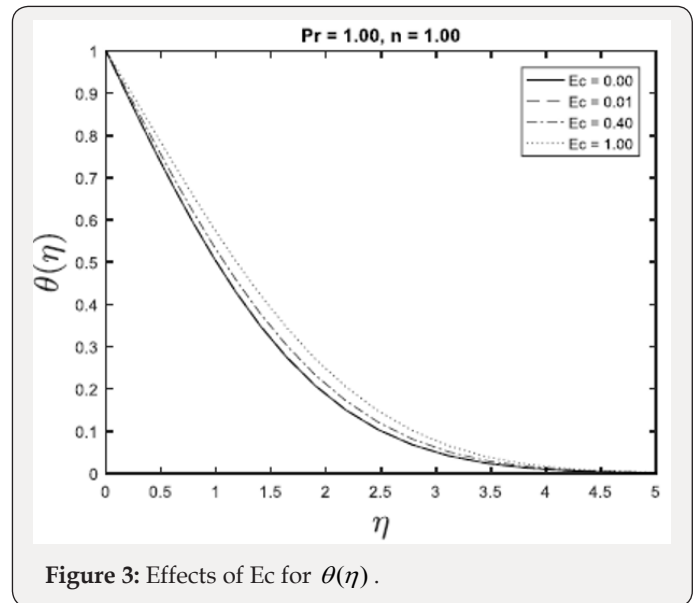
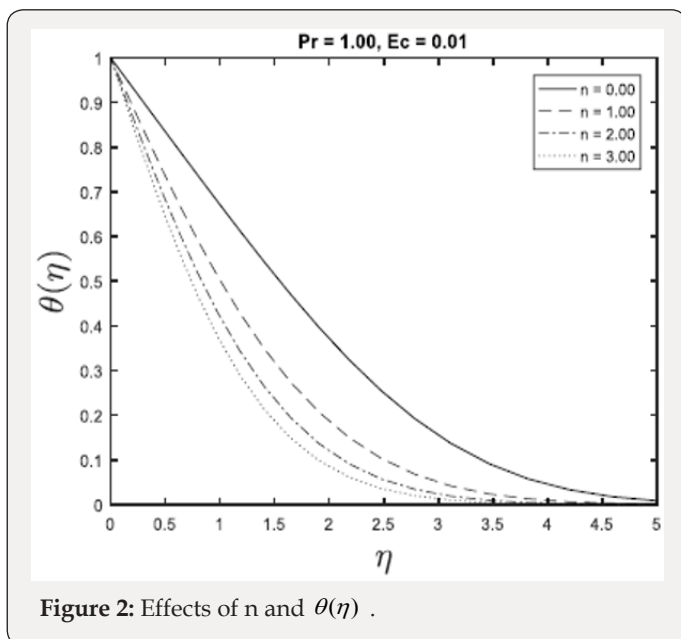
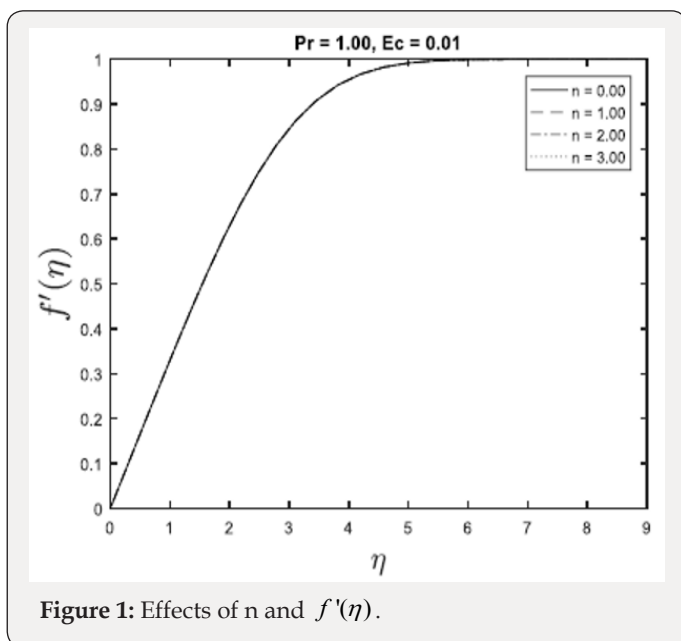
Comparison of numerical values of $f(\eta)$ with Ref: [16] when, $n = \text{Ec} = 0, \text{Pr} = 1$ (Table 4).

Table 4: Comparison of numerical values of $f(\eta)$ with Ref: [16] when, $n = \text{Ec} = 0, \text{Pr} = 1$.

η	Ref: [16]	Present work
0	0	0
0.2	0.066408	0.066407
0.4	0.132764	0.132764
0.6	0.198937	0.198940
0.8	0.264709	0.264707
1	0.329780	0.329774
1.2	0.393776	0.393768
1.4	0.456262	0.456265
1.6	0.516757	0.516783

This section concerns with the graphical illustrations obtained by using successive linearization method for velocity, temperature profiles. These profiles show the variations of embedded flow parameters in the solution expressions for heat transfer analysis for an incompressible viscous fluid. The physical interpretation of

the problem has been discussed in Figures 1 – 4. These figures are plotted in order to illustrate such variations. Here the graphs have been determined for the heat transfer flow of steady Newtonian fluid. Figures 1 & 2 shows the effects of the parameter on the velocity profile for $f'(\eta)$ and $\theta(\eta)$ when Ec, Pr are fixed. It is worth noticing that by increasing the parameter η reveals that buoyancy because of augments of gravity which boosts on the velocity. Figure 3 is sketched for the variation of Prandtl number Pr on $\theta(\eta)$. It is noted that for lager Pr , the thermal field is lower and then this reduce the temperature. In fact law Prandtl number Pr assist fluid with higher thermal conductivity and this create thicker thermal boundary layer than that for lager Pr . Finally, Figure 4 shows the effect of Ec on velocity and temperature profiles over the plate, and we note that by increasing in Ec parameter is seen that the effect is very big for the temperature.



Conclusion

In this research, the problem of heat transfer of an incompressible viscous fluid over flat pate is solved numerically. The numerical solutions are well established by SLM. The influence of various parameters is shown through different graphs. The present results have been tested and compared with the available published results in [16], in a limiting situation shown in tables v and an excellent agreement is found [17].

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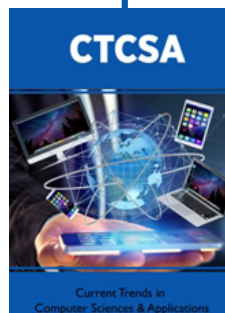
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