# Balanced Designs for Partial Double Cross 

Mahendra Kumar Sharma* and Mohammed Omer Ibrahim<br>Addis Ababa University, Ethiopia<br>*Corresponding author: Mahendra Kumar Sharma, Addis Ababa University, Ethiopia

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#### Abstract

Hinkelmann [1] constructed optimal balanced tetra partial cross i.e. optimal balanced partial double cross using balanced incomplete block designs and partially balanced incomplete block designs. In the present article we are presenting method of construction of three series of balanced partial double cross designs using nested balanced incomplete block designs in one way set up, when p is odd. Our proposed designs for partial double cross are new. AMS classification: 62 k 05.


Keywords: Nested Balanced Incomplete Block Design; Complete diallel Cross; Balanced Partial Double Cross; Block Design; Optimality

## Introduction

Mating designs involving multi-allele cross $m(\geq 2)$ ines play very important role to study the genetic properties of a set of inbred lines in plant breeding experiments. A most common type of mating design in genetics is the diallel cross which consist of $v=p(p-1) / 2$ crosses of p inbred lines such that the crosses of the type $(i \times j)=(j \times i)$ for $i, j=1,2, \ldots, p$. This type of mating design is called complete diallel cross (CDC) method 4 of Griffing (1956). The concept of CDC can be easily extended to double cross designs. A double cross mating design is obtained by crossing two unrelated $\mathrm{F}_{1}$ hybrids symbolized as $(i \times j)$ and $(k \times l)$, where $i \neq j \neq k \neq l$, are 4 parents and $(i \times j)$ and $(k \times l)$, are two $\mathrm{F}_{1}$ 's\{see Singh et al.2012\}. Omitting reciprocal crosses, there exist $3\left(\begin{array}{l}p \\ 4 \\ 4\end{array}\right)=\frac{p(p-1)(p-2)(p-3)}{8}$ different possible crosses from $p$ lines. We shall consider the case where one observes only a sample of all possible crosses. Such a design we call a partial tetra-allel cross (PTAC) or partial double cross.

Let $n_{c}$ denote the total number of crosses involved in a balanced partial double cross design. Generally double cross experiments are conducted using a Completely Randomized Design (CRD) or a Randomized Complete Block (RCB) design involving some or all of the $n_{c}$ crosses as treatments. The number of crosses in such a mating design increases rapidly with increase in the number of lines. It leads to an overall inefficient experiment. It is for this reason that the use of incomplete block design as environment design is needed for double cross experiments [2]. Constructed optimal block designs for partial double cross experiment by using balanced incomplete block designs and nested balanced incomplete
block designs [3]. Sharma and [4] constructed double cross designs for even and odd value of $p$ by using initial block of unreduced balanced incomplete block designs given [1] and initial block of row-column designs given by [5], respectively. In this paper we are using initial blocks of nested balanced incomplete block designs [6] [7] for the construction of three series of balanced partial double cross designs in one way set up, when $p$ is odd. The parameters of our proposed balanced partial double cross designs are different from [2] designs. In our proposed designs every cross is replicated equal number of times in a design and every line occurs in single cross with every line. We have considered the model that includes the gca effects, apart from block effects, but no specific combining ability effects.

## Some Definitions: Before we describe the constructions, it might be appropriate to recall some definitions.

Definition: The double cross has been defined [8] as a cross between two unrelated $F_{1}$ hybrids, say denoted by $(i \times j)$ and $(k \times l)$ where $i \neq j \neq k \neq l \neq i$, are denoting the grandparents and no two of them are same. Ignoring reciprocal crosses, with $p$ grandparents, there will be $3\binom{p}{4}$ different possible double crosses from $p$ lines. The number of combinations of 4 lines from p lines is $\binom{p}{4}=p p_{1}, p_{2} p_{3} / 24$, where $\mathrm{p}_{\mathrm{i}}$ denotes ${ }^{(p-i)}$. For each combination, three different double crosses can be formed by changing the manner in which the four lines are paired to form $F_{1}$ hybrid. We shall consider here the design where one observes only a sample of all possible crosses. Such a design we call a partial double cross (PDC).

Definition: According to [9], a set of mating is said to be partial double crosses, if the following conditions are satisfied:
a) Every line occurs exactly $r$ times in the set,
b) Every four-way cross occurs either once or not at all.

Definition: According to [9], a set of mating is said to be partial double crosses in the strict sense, if it satisfies the condition of Definition 2.2 and
a) Every single cross occurs once or not at all in the set.

Definition: According to [9], a partial double cross is said to be balanced partial double crosses, if every line occurs equally often with every other line in the same single cross.

Definition: According to [10], an arrangement of $p$ treatments each replicated $r^{*}$ times in two systems of blocks is said to be a nested balanced incomplete design (NBIBD) with parameters $\left(p, b_{1}, k_{1}, r^{*}, \lambda_{1}, b_{2}, k_{2}, \lambda_{2}, m\right)$ if
a) The second system is nested within the first, with each block from the first system, called hence forth as' block' containing exactly $m$ blocks from the second system, called hereafter as sub blocks.
b) Ignoring the second system leaves a balanced incomplete block design with parameters $p, b_{1}, k_{1}, r^{*}, \lambda_{1}$;
c) Ignoring the first system leaves a balanced incomplete block design with parameters $p, b_{2}, k_{2}, r^{*} h_{2}$. The following parametric relations hold for a nested balanced incomplete block design:
(i) $p r^{*}=b_{1} k_{1}=m b_{1} k_{2}=b_{2} k_{2},(i i)(p-1) \lambda_{1}=(k 1-1) r^{*},(i i i)(p-1) \lambda_{2}=(k 2-1) r^{*}$.

Definition: A design $d^{*} \varepsilon \mathrm{D}(p, b, k)$ is said to be A-optimal if and only if $\left(v_{i}\right) \leq \operatorname{strace}\left(V_{d}\right)$ means that trace of variance-covariance matrix $d^{*}$ is less than equal to trace of variance-covariance matrix $d$.

## Method of Construction

Consider a nested balanced incomplete block design $d$ obtained by developing $\mathrm{t}_{1}$ initial blocks mod (p), each sub-block is divided into $t_{2}$ sub-blocks. The parameters of such a NBIB design are $p, b_{1}=p t_{1}, b_{2}=b_{1} t_{2}, k_{1}=2 t_{2}, r^{*}, k_{2}=2$. If we identify the treatments of a design $d$ as lines of a diallel experiment and perform double crosses among the lines appearing in the same sub block of $d$ and arrange these sub blocks into one bigger block and develop this bigger block $\bmod (p)$, we get a design $d_{1}$ for a partial double cross experiment involving $p$ lines with $\mathrm{b}_{1 / 2}$ crosses arranged in p blocks. Each partial double cross is replicated once. Such a designd $d_{1}$ belongs to $\mathrm{D}(\mathrm{p}, \mathrm{b}, \mathrm{k})$.

For such a design $n_{d^{*} i j k l}=0$ or 1 for $i \neq j \neq k \neq l$,
The information matrix for designd $d_{1}$ is given below.
$c_{c_{11}}=(p-2)\left[I_{p}-\frac{\left(P^{2}-4 P+1\right)}{2 P(P-2)}{ }^{1 p_{p} 1_{p}}\right]$ (3.1)
$c_{a 1}$ given above by (3.1) is completely symmetric.
trace of $C_{d 1}=(p-1)(p-2)(3.2)$

Using $d_{1}$ each elementary contrast among line effects is estimated with a variance

$$
\frac{2 \sigma^{2}}{(p-2)}(3.3)
$$

Now we give the method of construction of three series of balanced partial double cross designs using NBIB designs [11].

$$
p=4 t+1, t \geq 1
$$

Series: Let $p=4$ be a prime or a prime power and $x$ be a primitive element of the GF ( $p$ ). Consider the following $t$ initial blocks.

$$
\left\{\left(x^{i}, x^{i+2 t}\right),\left(x^{i+t}, x^{i+3 t}\right)\right\}, i=0,1,2, \ldots t-1
$$

As shown [11], these initial blocks, when developed in the sense of Bose (1939), give rise to a nested balanced incomplete block design with parameters.
$v=p=4 t+1, b 1=t(4 t+1), r=4 t, k 1=4, \lambda 1=3, b 2=2 t(4 t+1) k 2=2, \lambda 2=1$
Arrange the following $m$ initial blocks into single block

$$
\left[\begin{array}{l}
\left(x^{i}, x^{i+2 t}\right) \\
\left(x^{i+1}, x^{i+3 i}\right)
\end{array}, i=0,1,2, \ldots, t-1\right]
$$

By making pair of crosses in a single block and developing mod ( $p$ ), we get balanced partial double
cross design with parameters

$$
p=4 t+1, b=, k=\frac{b_{2}}{t(4 t+1)}
$$

Remark: For construction of balanced partial double cross design it is necessary that the block size of a single block must be an even number i.e. $t$ must be a multiple of 2 . The procedure to construct the above designs, has been explained by the following illustrative examples.

Example: If we let $t=2$, we get the following two blocks.

$$
\left[\begin{array}{cc}
(1,2) & (x, 2 x) \\
(2 x+1, x+2) & (2 x+2, x+1)
\end{array}\right]
$$

We can now write both blocks in a single block as given below.

$$
\left[\begin{array}{c}
(1,2) \\
(2 x+1, x+2) \\
(x, 2 x) \\
(2 x+2, x+2)
\end{array}\right]
$$

where is a primitive element of $\mathrm{GF}\left(3^{2}\right)$. Now cross the elements of the individual block and put these crosses in a single block. Adding successively the non-zero elements of GF ( $3^{2}$ ) to the contents of the single block, we obtain block design for partial double cross experiment with parameters $p=9, b=9, k=2$. The design is exhibited below, where the lines have been relabelled 1-9, using the correspondence $0 \rightarrow 1,1 \rightarrow 2,2 \rightarrow 3, x \rightarrow 4, x+1 \rightarrow 5, x+2 \rightarrow 6,2 x+1 \rightarrow 8,2 x+2 \rightarrow 9:$

## Partial Double Cross Block Design

$$
\begin{aligned}
& \mathbf{B}_{1} \mathbf{B}_{2} \mathbf{B}_{3} \mathbf{B}_{4} \mathbf{B}_{5} \mathbf{B}_{6} \\
& (2 \times 3) \times(6 \times 8)(1 \times 3) \times(4 \times 9) \quad(1 \times 2) \times(5 \times 7)
\end{aligned}
$$

$\begin{array}{lll}(5 \times 6) \times(2 \times 9) & (4 \times 6) \times(3 \times 7) & (4 \times 5) \times(1 \times 8) \\ (4 \times 7) \times(5 \times 9) & (5 \times 8) \times(6 \times 7) & (6 \times 9) \times(4 \times 8) \quad(1 \times 7) \times(3 \times 8) \\ (2 \times 8) \times(1 \times 9) & (3 \times 9) \times(2 \times 7) & \\ \mathbf{B}_{7} \text { B }_{8} \text { B }_{9} & \\ (8 \times 9) \times(3 \times 5)(7 \times 9) \times(1 \times 6) & (7 \times 8) \times(2 \times 4) \\ (1 \times 4) \times(2 \times 6)(2 \times 5) \times(3 \times 4) & (3 \times 6) \times(1 \times 5)\end{array}$
According to [2] the above design is balanced and is of minimal size, hence optimal.

Series: Let $p=6 t+1, t \geq 1$ be a prime or a prime power and $x$ be a primitive element of the Galois field of order $p$. Consider the set of initial blocks

$$
\left\{\left(x^{i}, x^{i+3 t}\right),\left(x^{i+t}, x^{i+4 t}\right),\left(x^{i+2 t}, x^{i+5 t}\right)\right\}, i=0,1,2, \ldots t-1 .
$$

Dey, et al. (1986) showed when these initial blocks developed over mod (p), give a solution of a nested incomplete block design with parameters $p=6 t+1, b_{1}=t(6 t+1), r=6 t, k_{1}=6, \lambda_{1}=5, b_{2}=3 t(6 t+1), k_{2}=2, \lambda_{2}=1$.

We can then arrange the above initial blocks into a single block as given below

$$
\left[\begin{array}{l}
\left(x^{\prime}, x^{t+1 \pi}\right) \\
\left(x^{+2+}, x^{2+1+1}\right),=0,1, \ldots, t-1 \\
\left(x^{2+2}, x^{4 s t}\right)
\end{array}\right]
$$

Example 2: Let $t=2$. Then we get the following two initial blocks.

$$
\left[\begin{array}{cc}
(1,12) & (2,11) \\
(4,9) & (8,5) \\
(3,10) & (6,7)
\end{array}\right]
$$

We can arrange these two blocks in a single block as given below.

$$
\left[\begin{array}{c}
(1,12) \\
(4,9) \\
(3,10) \\
(2,11) \\
(8,5) \\
(6,7)
\end{array}\right]
$$

Now performing crosses in pairs and developing these crosses over $\bmod (p)$, we obtain A- optimal block
design for partial double cross with parameters

$$
p=13, b=13 \text { and } k=3 \text {. }
$$

Series: Let $p=2 t+1, t \geq 2$ be a prime or a prime power and consider the following $m$ blocks

$$
(0,2 t),(1,2 t-1),(2,2 t-1) \ldots . .(t-1, t+1) \bmod (2 t+1)
$$

Example 3: If we let $t=4$, then the single block will be as given below.
$\left[\begin{array}{c}(0,8) \\ (1,7) \\ (2,5) \\ (3,5)\end{array}\right]$

Now applying the procedure given above in example 1, we can obtain an A-optimal block design for double cross experiment with parameters

$$
p=9, b=9, k=2
$$

Note 2: The t blocks given in series 3 form a nested balanced incomplete block design with parameters

$$
p=2 t+1, b_{1}=t(2 t+1), r=4 k_{1}=4, \lambda_{1}=6, b_{2}=2 t(2 t+1), k_{2}=2, \lambda_{2}=2
$$

given [10].

## Analysis of Bpdc Design

Following [9] we can assume the following reduced model for proposed incomplete balanced partial double cross (BPDC) mating designd $d_{1}$. For the $\operatorname{BPDC}(i \times j)(k \times l)$, where $i \neq j \neq k \neq l$, in m blocks, we adopt the following model

$$
y_{(i j)(k l)}=\mu+g_{i}+g_{j}+g_{k}+g_{l}+\beta_{m}+e_{i j k l m}
$$

where $y_{(i j)(l)}$ is the observation on double cross $(i j)(k l)$ from m blocks, $m=1,2, \ldots, p, i, j, k, l=1,2, \ldots, p ; i \neq j \neq k \neq l, \mu$ is the general mean, $g_{i}$ is the effect of ith line, $\beta_{\mathrm{m}}$ is the effect mthblock and $e_{i j l d m}$ are random errors assumed independent and normally distributed with mean zero and variance $\sigma^{2}$.

An incomplete four - way mating design $d_{1}$ is defined such that every line occurs exactly $r$ times and every four-way cross occurs once or not all [9]. In matrix notation (4.1) can be written as below

$$
y=\mu 1_{n}+\Delta_{1}^{\prime} g+\Delta_{2}^{\prime} \beta+e_{(4.2)}
$$

$\mathbf{y}$ is an $\mathrm{n} \times 1$ vector of observations, 1 is an $\mathrm{n} \times 1$ vector of ones, $\ddot{\mathbf{A}}_{1}^{\prime}$ is an $\mathrm{n} \times \mathrm{p}$ design matrix for lines and $\widetilde{\mathbf{A}}_{2}^{\prime}$ is an $\mathrm{n} \times \mathrm{b}$ design matrix for blocks, that is, the $(h, l)^{\text {th }}$ element of $\widetilde{\mathbf{A}}_{1}^{\prime}\left(\right.$ also of $\left.\dddot{\mathbf{A}}_{2}^{\prime}\right)$ is 1 if the $h^{\text {th }}$ observation pertains to the $l^{\text {th }}$ line (also of block) and is zero otherwise. $\mu$ is a general mean, $g$ is a $p \times 1$ vector of line parameters, $\boldsymbol{\beta}$ is a $p \times 1$ vector of block parameters and $\mathbf{e}$ is an $n$ $\times 1$ vector of residuals. It is assumed that vector $\boldsymbol{\beta}$ is fixed and $\mathbf{e}$ is normally distributed with mean $\mathbf{0}$ and $\operatorname{Var}(\mathbf{e})=\sigma^{2}$ IwithCov $(\boldsymbol{\beta}, \mathbf{e})$ $=\mathbf{0}$, where $\mathbf{I}$ is the identity matrix of conformable order. For the analysis of proposed design $d_{1}$, the method of least squares leads to the following reduced normal equations for estimating the linear functions of the line effects of lines under model (4.1).

$$
\begin{aligned}
& C_{d 1} \hat{g}=Q_{d 1} \text { (4.3) } \\
& \text { where } C_{d 1}=G_{d 1}-N_{d 1} K_{d 1}^{-1} N_{d 1}^{\prime}=(p-2)\left[I_{P}-\frac{\left(P^{2}-4 P+1\right)}{2 P(P-2)} 1^{P} 1_{P}^{\prime}\right](i, j=1,2, \ldots, p) \text { (4.4) } \\
& \begin{array}{l}
\text { and } \hat{\hat{g}^{\prime}}=\left(\left(\hat{g}_{1}, \hat{g}_{2}, \ldots, \hat{g}_{p}\right) \text { and } Q_{d 1}^{\prime}=\left(Q_{1}, Q_{2}, \ldots \ldots \ldots Q_{P}\right)\right. \\
\text { and } \\
Q_{i}=T_{t}-K_{d}^{-1} \sum_{j=1}^{p} n_{d i} B_{j}, i=1,2, \ldots, \text { pandj }=1,2, \ldots, p
\end{array} \\
& \text { where } G_{d 1}=\Delta_{1} \Delta_{1}^{L_{1}}=\left(g_{d i f}\right), g_{d|i| i}=s_{d i t} \text { and for } i \neq i, g_{d 1 i i} \text { is the number of crosses }
\end{aligned}
$$ in d in which the lines $i$ and $i^{\prime}$ appear together. $N_{d 1}=\Delta_{1} \Delta_{2}^{\prime}=(n d 1 i j), n d 1 i j$ is the number of times the line $i$ occurs in block $j$ of $d_{1}$ and $K_{d 1}=\Delta_{2} \Delta_{2}^{\prime}$ is the diagonal matrix of block sizes? $\mathrm{T}_{\mathrm{i}}$ is the total of $\mathrm{i}^{\text {th }}$ line in design $d_{1}$.

The sum of the elements in each row and each column of $\mathbf{C}_{\mathrm{d} 1}$ is equal to zero. Hence rank $\left(C_{d 1}\right) \leq p-1$ but for all differences $g_{i}-g_{j}$, to be
estimable it is necessary that rank $\left(C_{d 1}\right)=p-1$. A solution of (4.3) is then given by

$$
\hat{g}=C_{d 1}^{-} Q_{d 1} \text { (4.7) }
$$

where $C_{i n}=\frac{1}{(p-2)}\left[I_{n}-\frac{\left(p^{p}-4 p+1\right)}{P(P-1)(P-5) r^{1} r_{1},}\right.$, is the generalized inverse of $C_{\mathrm{d} 1}$ with property $C_{d 1} C_{d 1}^{-} C_{d 1}=C_{d 1}$. The sum of squares due to crosses is


$$
E\left(Q_{d 1}\right)=C_{d 1} \text { gand } \operatorname{var}\left(Q_{d 1}\right)=\sigma^{2} C_{d 1} \text { (4.8) }
$$

From (4.7), it follows immediately that the using $d_{1}$ each elementary contrast among line effects is estimated with a variance

$$
\operatorname{var}\left(\hat{g_{i}}-\hat{g_{j}}\right)=\frac{2 \sigma^{2}}{(p-2)}
$$

Now let us suppose that the proposed design $d_{1}$ is replicated $u$ times because if we analysis the proposed design $\mathrm{d}_{1}$ no degrees of freedom is left for the error. Now we consider the following model for replicated BPDC design.

$$
y_{(i j)(k i)^{v}}=\mu+y_{i j l l}^{*}+r_{v}+e_{i j k v} \text { (4.10) }
$$

where $y(i j)(k l) v$ is the observation on double cross $(i j)(k l)$ from $v^{\text {th }}$ replicate?

$$
v=1,2, \ldots, u, i, j, k, l=1,2, \ldots, p ; i \neq j \neq k \neq l \neq, \mu \text { is the }
$$ general mean,

$y^{*}(i j)(k l)=\mu+g_{i}+g_{j}+g_{k}+g_{l}+\beta_{m}+e_{i j k m}, r v$ is the effect $\mathrm{v}^{\text {th }}$ replicate and $e_{\text {epsat }}$ are random errors assumed independent and normally distributed with mean zero and variance $\sigma_{\varepsilon}^{2}$. The Analysis of variance of replicated BPDC design is given in Table 1.

Table 1: Analysis of variance of PTAC.

| Source | D.F. | S.S. | E(M.S) |
| :---: | :---: | :---: | :---: |
| Replication | u-1 | $\frac{1}{p k} \sum_{k=1}^{u} R_{k}-\frac{G^{2}}{u p k}$ |  |
| Blocks with in replication | $\mathrm{u}(\mathrm{p}-1)$ | $\frac{1}{k} \sum_{j=1}^{p} B_{J}-\frac{G^{2}}{u p k}$ |  |
| Crosses | pk-1 | $Q_{d 1}^{\prime} C_{d 1}^{-} Q_{d 1}$ |  |
| GCA | p-1 | $\sum_{i=1}^{p} \hat{g_{i}} Q_{i}$ | $\sigma_{e}^{2}+u \sigma^{2}+u(p-1) \sigma_{g}^{2}$ |
| Interactions | pk-p | Difference | $\sigma_{e}^{2}+u \sigma^{2}$ |
| Reminder | $(\mathrm{pk}-1)(\mathrm{u}-1)-(\mathrm{p}-2)$ | Difference | $\sigma^{2}$ |
| Total | upk -1 | $\sum_{s} \sum y_{(i j)(k l) s}-\frac{G^{2}}{u p k}$ |  |

## Optimality

A design $d_{1}$ is connected since rank $\left(C_{d 1}\right)=p-1$ and variance balanced because the matrix $\mathbf{C}_{\mathrm{d} 1}$ is completely symmetric. It implies that all elementary comparisons among line effects are estimable using $d_{1}$ with the same variance. We denote by $\mathrm{D}(\mathrm{p}, \mathrm{b}, \mathrm{k})$, the class of all such connected block design $\{d\}$ with $p$ lines, $b$ blocks each of size $k$. According to [9], such a design can be considered as being optimal in the sense that it is completely balanced and of minimal size. So, our proposed design $d_{1}$ is optimal and of minimal size. Now we will use the Kiefer criteria to see the status of our proposed design $d_{1}$ for universal optimality.

Kiefer (1975) showed that a design is universally optimal in a relevant class of competing design:
a) The information matrix ( the $\mathbf{C}_{\mathrm{d}}$ - matrix) of the design is completely symmetric in the sense that $\mathbf{C}_{d}$ has all its diagonal elements equal and all of its off- diagonal elements equal, and
b) The matrix $\mathbf{C}_{\mathrm{d}}$ has maximum trace in the class of competing designs, that is, such a design minimizes the average variance of the best linear unbiased estimators of all elementary contrasts among the parameters of interest i.e. the line effects in our context. Now we state the Kiefer criterion of the universal optimality in sense [2] to compare proposed designs optimality with universal optimal designs.

Theorem: For any proposed design $d^{*} \varepsilon D(p, b, k)$ be a block design for m -allelcrosses satisfying
a) Trace $\left(C_{d^{*}}\right)=K^{-1} b\{m k(k-1-2 x)+p x(x+1)\}$
b) $\quad C_{d^{*}}=(p-1)^{-1} k^{-1} b\{m k(k-1-2 x)+p x(x+1)\}\left(I P-1 P 1^{\prime} P / P\right)$ is completely symmetric.

Where $x=[2 k / p]$ where $[\mathrm{z}]$ denotes largest integer not exceeding z , Ip is an identity matrix of order p and $\mathbf{1 p} \mathbf{1}^{\prime} \mathrm{p}$ is a $\mathrm{p} \times \mathrm{p}$ matrix of all ones. Furthermore, using $d^{*}$ all elementary contrasts among line effects are estimated with variance

$$
\left[2 b^{-1}(p-1) K /\{m k(k-1-2 k)+p x(x+1)\}\right] \sigma^{2} .
$$

Then $d^{*}$ is universally optimal in $\mathbf{D}(\mathrm{p}, \mathrm{b}, \mathrm{k})$, and in particular minimizes the average variance of the best linear unbiased estimator of all elementary contrasts among the line effects. According to the definition 2.6, the trace of our proposed design
$d_{1}=(p-1)(p-2)$ is greater than the equality given in theorem 4.1 i.e., $4 p=36$. Hence our design $d_{1}$ is also A-optimal. Hence, we state the following theorems.

Theorem: The existence of a nested balanced incomplete block design $d$ with parameters

$$
p, b_{1}=p t_{1}, b_{2}=b_{1} t_{2}, k_{1}=2 t_{2}, r^{*}, k_{2}=2 \text {, where } p \text { is odd, implies the }
$$ existence of A-optimal block designs for balanced partial double cross experiment.

## Discussion

A-Optimal balanced partial double cross designs with $p \leq 16, s \leq 30$ obtained by the above methods from NBIB designs [7], are listed in the following Table 2. These are the designs other than the designs catalogued $[12,10,8]$. These are the new designs and can be used in agricultural experimentation successfully $[13,14]$.

Table 2: optimal block design for double cross with $\mathrm{p} \leq 16, \mathrm{~s} \leq 30$ generated by using NBIB designs of Morgan [3].

| S.No. | $\mathbf{p}$ | $\mathbf{k}$ | $\mathbf{b}$ | Fractionation | Source |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 9 | 9 | 2 |  | MPR 5w |
| 2. | 9 | 9 | 2 |  | MPR8 |
| 3. | 11 | 11 | 5 | MPR49 |  |
| 4. | 13 | 13 | 3 | MPR20w |  |
| 5. | 13 | 13 | 3 | MPR 21 |  |
| 6. | 15 | 13 | 3 |  | MPR 23 |
| 7. | 13 | 15 | 39 | 7 | MPR33w |
| 8. | 15 | 2 |  | MPR59 |  |
| 9 | 10 |  |  | 3 |  |

## Conclusion

We have presented a method of construction of three series of A-optimal balanced block designs for partial double cross by using nested balanced incomplete block designs. Our proposed Aoptimal block designs for partial double crosses are new.

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